Inverse Functions

SCAN ME

REVISE THIS TOPIC

ANSWERS

1 $f(x) = 2x + 9$ $g(x) = \sqrt{x-3}$ $h(x) = x^3 + 4$

1 (a) Work out $f^{-1}(x)$ **[2 marks]**

$y = 2x + 9$

$x = 2y + 9$

$x - 9 = 2y$

$\frac{x-9}{2} = y$

$f^{-1}(x) = \frac{x-9}{2}$

1 (b) Work out $g^{-1}(x)$ **[2 marks]**

$y = \sqrt{x-3}$

$x = \sqrt{y-3}$

$x^2 = y - 3$

$x^2 + 3 = y$

$g^{-1}(x) = x^2 + 3$

1 (c) Work out $h^{-1}(31)$ **[2 marks]**

$y = x^3 + 4$

$x = y^3 + 4$

$x - 4 = y^3$

$\sqrt[3]{x-4} = y$

$h^{-1}(x) = \sqrt[3]{x-4}$

$h^{-1}(31) = \sqrt[3]{31-4}$

Answer 3

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2 $f(x) = \frac{2x + 3}{4}$ $g(x) = x^2 - 6$

2 (a) Work out $f^{-1}(x)$ **[2 marks]**

$y = \frac{2x+3}{4}$ $4x = 2y + 3$ $4x - 3 = 2y$ $\frac{4x-3}{2} = y$

$x = \frac{2y+3}{4}$

$f^{-1}(x) =$ $\frac{4x-3}{2}$

2 (b) Work out $g^{-1}(x)$ **[2 marks]**

$y = x^2 - 6$ $\sqrt{x+6} = y$

$x = y^2 - 6$

$x + 6 = y^2$

$g^{-1}(x) =$ $\sqrt{x+6}$

3 $f(x) = 50 - x^2$ $g(x) = 4x^2 - 1$

3 (a) Work out $f^{-1}(1)$ **[2 marks]**

$y = 50 - x^2$ $y^2 = 50 - x$

$x = 50 - y^2$ $y = \sqrt{50-x}$

$x + y^2 = 50$ $f^{-1}(1) = \sqrt{50-1}$

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Answer

3 (b) Work out $g^{-1}(0)$ **[2 marks]**

$y = 4x^2 - 1$ $\frac{x+1}{4} = y^2$ $g^{-1}(x) = \sqrt{\frac{x+1}{4}}$

$x = 4y^2 - 1$ $\sqrt{\frac{x+1}{4}} = y$ $g^{-1}(0) = \sqrt{\frac{0+1}{4}}$

$x + 1 = 4y^2$

$\frac{1}{2}$

Answer

4 $f(x) = \frac{2x^2}{5}$ $g(x) = \frac{x}{4} - 3$

4 (a) Work out $f^{-1}(x)$ **[2 marks]**

$y = \frac{2x^2}{5}$ $5x = 2y^2$ $y = \sqrt{\frac{5x}{2}}$

$\frac{5x}{2} = y^2$

$x = \frac{2y^2}{5}$

$f^{-1}(x) =$ $\sqrt{\frac{5x}{2}}$

4 (b) Work out $g^{-1}(x)$ **[2 marks]**

$y = \frac{x}{4} - 3$ $x + 3 = \frac{y}{4}$

$x = \frac{y}{4} - 3$ $4(x+3) = y$

$g^{-1}(x) =$ $4(x+3)$

5 $f(x) = \sqrt[3]{100 - x}$ $g(x) = 2(x + 14)$

5 (a) Work out $f^{-1}(4)$ **[2 marks]**

$y = \sqrt[3]{100 - x}$ $x^3 + y = 100$

$x = \sqrt[3]{100 - y}$ $y = 100 - x^3$

$x^3 = 100 - y$ $f^{-1}(4) = 100 - 4^3$

Answer 36

5 (b) Work out $g^{-1}(26)$ **[2 marks]**

$y = 2(x+14)$ $\frac{x}{2} - 14 = y$

$x = 2(y+14)$ $g^{-1}(26) = \frac{26}{2} - 14$

$\frac{x}{2} = y + 14$

Answer -1

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6 $f(x) = \frac{5}{x+10}$ $g(x) = \sqrt{2x^3 - 3}$

6 (a) Work out $f^{-1}(x)$ **[2 marks]**

$y = \frac{5}{x+10}$ $x(y+10) = 5$

$x = \frac{5}{y+10}$ $y+10 = \frac{5}{x}$

$y = \frac{5}{x} - 10$

$f^{-1}(x) =$ $\frac{5}{x} - 10$

6 (b) Work out $g^{-1}(x)$ **[2 marks]**

$y = \sqrt{2x^3 - 3}$ $x^2 + 3 = 2y^3$ $y = \sqrt[3]{\frac{x^2+3}{2}}$

$x = \sqrt{2y^3 - 3}$ $\frac{x^2+3}{2} = y^3$

$x^2 = 2y^3 - 3$

$g^{-1}(x) =$ $\sqrt[3]{\frac{x^2+3}{2}}$

7 $f(x) = 3 - \frac{2}{x}$ $g(x) = (x-5)^3$

7 (a) Work out $f^{-1}(2.5)$ **[2 marks]**

$y = 3 - \frac{2}{x}$ $x + \frac{2}{y} = 3$ $\frac{2}{3-x} = y$

$x = 3 - \frac{2}{y}$ $\frac{2}{y} = 3 - x$ $f^{-1}(2.5) = \frac{2}{3-2.5}$

Answer 4

7 (b) Work out $g^{-1}(27)$ **[2 marks]**

$y = (x-5)^3$ $\sqrt[3]{x} + 5 = y$

$x = (y-5)^3$ $g^{-1}(27) = \sqrt[3]{27} + 5$

$\sqrt[3]{x} = y - 5$

Answer 8

8 $f(x) = \frac{x+4}{x-3}$ $g(x) = \sqrt{3x}$ $h(x) = 2x + 1$

8 (a) Work out $f^{-1}(x)$ **[3 marks]**

$y = \frac{x+4}{x-3}$ $x(y-3) = y+4$ $y = \frac{4+3x}{x-1}$

$xy - 3x = y + 4$

$x = \frac{y+4}{y-3}$ $xy - y = 4 + 3x$

$y(x-1) = 4 + 3x$

$f^{-1}(x) = \frac{4+3x}{x-1}$

8 (b) Work out $g^{-1}(9)$ **[2 marks]**

$y = \sqrt{3x}$ $\frac{x^2}{3} = y$

$x = \sqrt{3y}$ $g^{-1}(9) = \frac{9^2}{3}$

$x^2 = 3y$

Answer 27

8 (c) $k(x) = gh(x)$
Work out $k^{-1}(x)$ **[4 marks]**

$gh(x) = \sqrt{3(2x+1)}$ $x = \sqrt{6y+3}$

$gh(x) = \sqrt{6x+3}$ $x^2 = 6y + 3$

$x^2 - 3 = 6y$

$k(x) = \sqrt{6x+3}$ $\frac{x^2-3}{6} = y$

$y = \sqrt{6x+3}$

$k^{-1}(x) = \frac{x^2-3}{6}$

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