Circle Theorems

1 *A*, *B*, *C* and *D* are points on the circumference of a circle with centre *O*.

Angle *ABD* = 81°
Angle *BDC* = 42°

(a) Work out the size of angle *CAB*.

42 °

(1)

(b) Work out the size of angle *ACD*.

81 °

(1)

(c) Work out the size of angle *AEB*.

57 °

(1)

(d) Work out the size of angle *BEC*.

123 °

(1)

(Total for Question 1 is 4 marks)

1

2 *A*, *B* and *C* are points on the circumference of a circle with centre *O*.
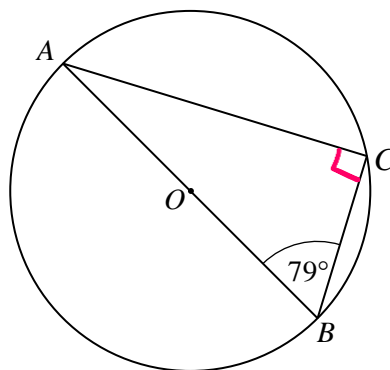
Angle *ABC* = 79°

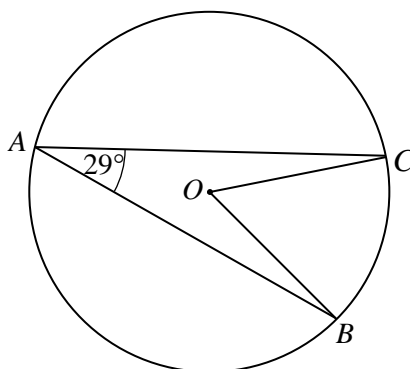(a) Work out the size of angle *CAB*.

_____ 11 _____ °

(1)

(b) Give a reason for your answer to part (a)

The angle in a semicircle is 90°
Angles in a triangle add to 180°

(1)

(Total for Question 2 is 2 marks)

3 *A*, *B* and *C* are points on the circumference of a circle with centre *O*.

Angle *CAB* = 29°

(a) Work out the size of angle *COB*.

_____ 58 _____ °

(1)

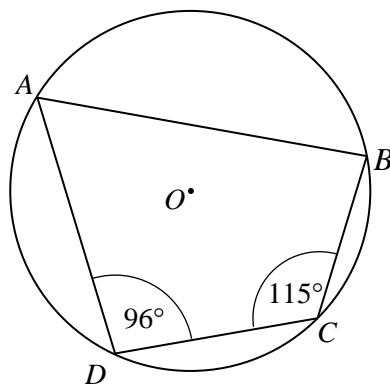(b) Give a reason for your answer to part (a)

The angle at the centre is twice the
angle at the circumference.

(1)

(Total for Question 3 is 2 marks)

2

4 *A*, *B*, *C* and *D* are points on the circumference of a circle with centre *O*.

Angle *ADC* = 96°
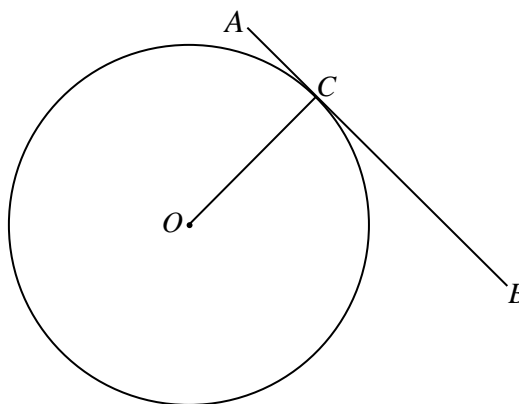Angle *BCD* = 115°

(a) Work out the size of angle *ABC*.

84 °

(1)

(b) Give a reason for your answer to part (a)

Opposite angles in a cyclic quadrilateral add to 180°

(1)

(Total for Question 4 is 2 marks)

5 *A*, *B*, and *C* are points on the circumference of a circle with centre *O*.
AB is the tangent to the circle at point *C*.

(a) Write down the size of angle *OCB*.

90 °

(1)
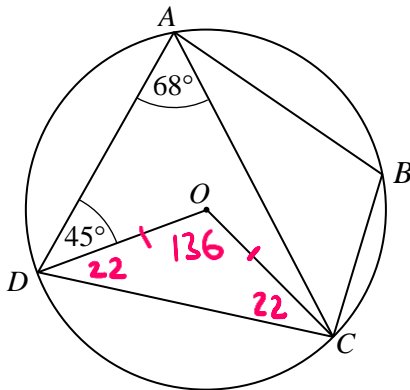
(b) Give a reason for your answer to part (a)

A tangent meets a radius at 90°

(1)

(Total for Question 5 is 2 marks)

3

6 *A*, *B*, *C* and *D* are points on the circumference of a circle with centre *O*.

Angle *DAC* = 68°
Angle *ADO* = 45°

Work out the size of angle *ABC*.
Give reasons for each stage of your working.

Angle DOC = 136°
The angle at the centre is twice the
angle at the circumference.
Angle ODC = Angle OCD = 22°
Base angles in an isosceles triangle are equal
Angle ADC = 22 + 45
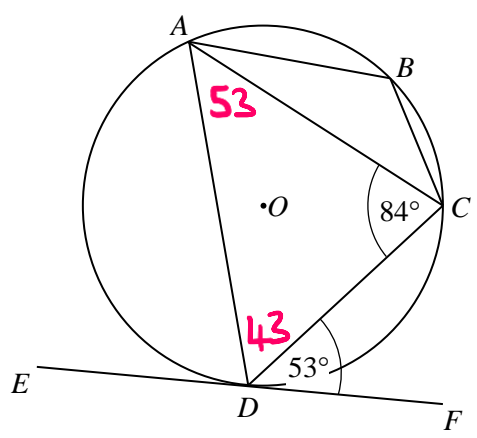   = 67
Angle ABC = 180 - 67
          = 113
Opposite angles in a cyclic quadrilateral
add to 180°

_____113_____ °

(Total for Question 6 is 4 marks)

1st

4

7 A, B, C and D are points on the circumference of a circle with centre O.
EF is the tangent to the circle at point D.

A

53

B

•O

84° C

43

53°

E

D

F

Angle ACD = 84°
Angle CDF = 53°

Work out the size of angle ABC.
Give reasons for each stage of your working.

Angle DAC = 53°
Alternate segment theorem
Angle ADC = 43°
Angles in a triangle add to 180°
Angle ABC = 180 - 43
         = 137°
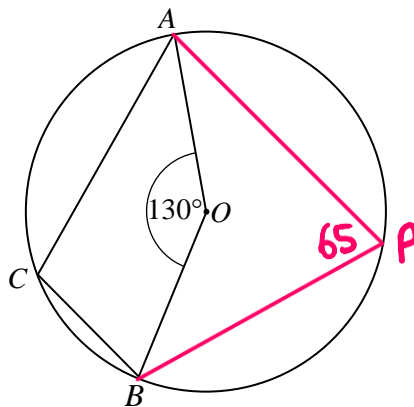Opposite angles in a cyclic quadrilateral
add to 180°

137 °

(Total for Question 7 is 4 marks)

5

8 *A*, *B*, and *C* are points on the circumference of a circle with centre *O*.

A
130° O
65 P
C
B

Angle *AOB* = 130°

Work out the size of angle *ACB*.
Give reasons for each stage of your working.

Angle APB = 65°
The angle at the centre is twice the
an le at the circumference.

Angle ABC = 180 - 65
= 115
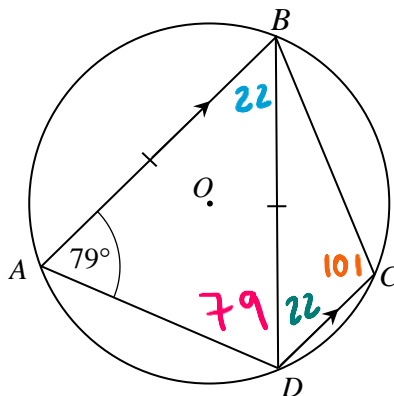Opposite angles in a cyclic quadrilateral
add to 180°

115 °

(Total for Question 8 is 3 marks)

6

9 *A*, *B*, *C* and *D* are points on the circumference of a circle with centre *O*.

Angle *BAD* = 79°
BA = *BD*
Lines *AB* and *DC* are parallel.

Work out the size of angle *DBC*.
Give reasons for each stage of your working.

Angle BAD = Angle BDA = 79°
Base angles in an isosceles triangle are equal

Angle ABD = 22°
Angles in a triangle add to 180°

Angle BDC = Angle ABD
Alternate angles are equal

Angle ABC = 180 − 43
= 137°
Opposite angles in a cyclic quadrilateral add to 180°

Angle DBC = 57°
Angles in a triangle add to 180°
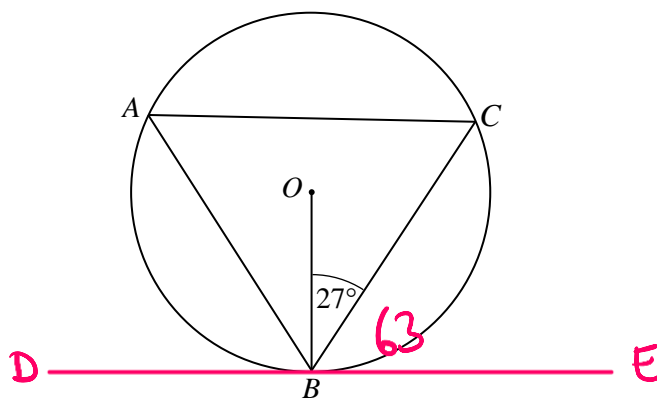
57 °

(Total for Question 9 is 5 marks)

1st

7

10 *A*, *B* and *C* are points on the circumference of a circle with centre *O*.

Angle *CBO* = 27°

Work out the size of angle *BAC*.
Give reasons for each stage of your working.

Angle CBE = 90 − 27
= 63°

A tangent meets a radius at 90°
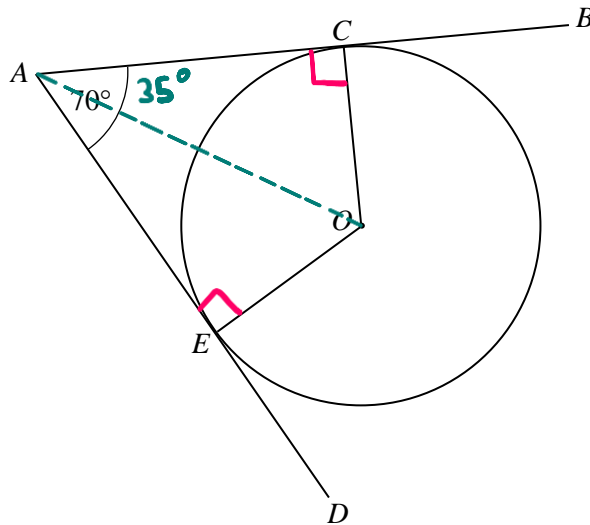
Angle BAC = 63°
Alternate segment theorem

63

(Total for Question 10 is 3 marks)

11 *C* and *E* are points on the circumference of a circle with centre *O*.

AB and *AD* are tangents to the circle at *C* and *E*.
Angle *EAC* = 70°

(a) Work out the size of angle *COE*.

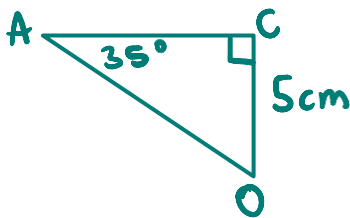Angle AEO = Angle ACO = 90°

A tangent meets a radius at 90°

Angle COE = 110°   Angles in a quadrilateral add to 360°

$OC = 5$ cm

_____110_____ °

(2)

(b) Work out the length of *CA*.
Give your answer to 1 decimal place.

Angle CAO = 35°

$\tan(35) = \dfrac{5}{CA}$

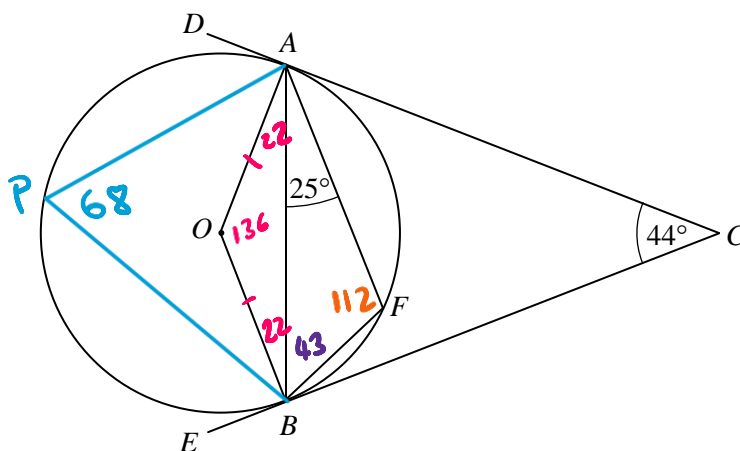$CA = \dfrac{5}{\tan(35)}$

$CA = 7.1407..$ ____7.1____ cm

(2)

(Total for Question 11 is 4 marks)

1st

9

12 A and B are points on the circumference of a circle with centre O.

[Diagram showing a circle with centre O, points A, B, D, E, P, F, C with angles marked: 22, 25°, 136, 68, 44°, 112, 22, 43]

DC and EC are tangents to the circle at A and B.

Angle ACB = 44°

Angle BAF = 25°

[Reasons not required in this Q]

Work out the size of angle FBC.

Angle AOB = 136° A tangent meets a radius at 90°
and angles in a quadrilateral add to 360°

Angle ABO = Angle BAO = 22°

Base angles in an isosceles triangle are equal

Angle APB = 68°

The angle at the centre is twice the angle at the circumference.

Angle AFB = 112°

Opposite angles in a cyclic quadrilateral add to 180°

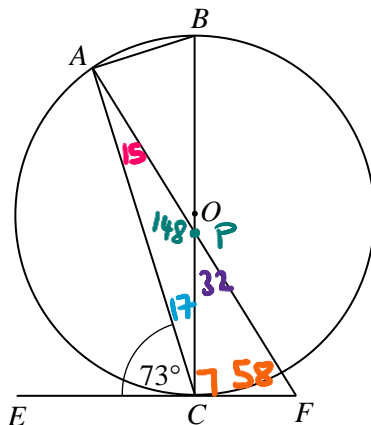Angle ABF = 43° Angles in a triangle add to 180°

Angle FBC = 25°
        A tangent meets a radius at 90°

___25___ °

(Total for Question 12 is 4 marks)

1st

10

13 A, B and C are points on the circumference of a circle with centre O.

EF is the tangent to the circle at *C*.
Angle $ACE = 73°$
Angle $FAB = 5 \times$ Angle CAF

Work out the size of angle AFC.

[Reasons not required in this Q]

Angle BAC = 90° The angle in a semicircle is 90°
90 ÷ 6 = 15
Angle CAF = 15°
Angle ECB = 90°   Angle ACB = 17°
A tangent meets a radius at 90°
Angle APC = 148° Angles in a triangle add to 180°
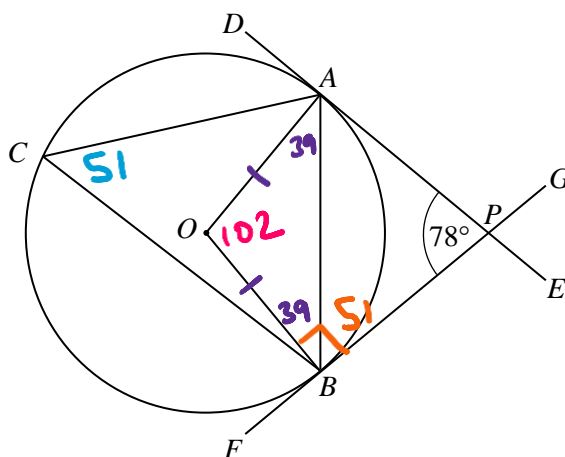Angle CPF = 32° Angles on a straight line add to 180°

Angle AFC = 58° Angles in a triangle add to 180°

58 °

(Total for Question 13 is 4 marks)

11

14 *A*, *B* and *C* are points on the circumference of a circle with centre *O*.

DE and *FG* are tangents to the circle at *A* and *B* that intersect at the point *P*.
Angle *APB* = 78°

[Reasons not required in this Q]

(a) Work out the size of angle *ACB*.

Angle AOB = 102° A tangent meets a radius at 90°
and angles in a quadrilateral add to 360°
Angle ACB = 51° Angle at the circumference is
half the angle at the centre

51 °

(2)

(b) Work out the size of angle *ABP*.

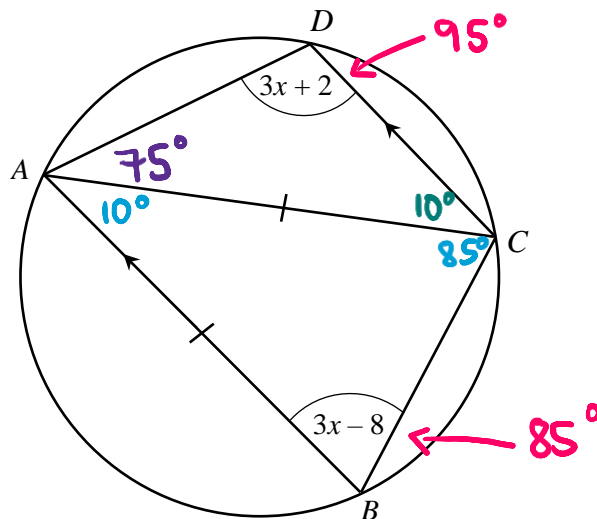Angle ABO = Angle BAO = 39°
Base angles in an isosceles triangle are equal

Angle ABP = 90 - 39
     = 51

A tangent meets a radius at 90°

51 °

(2)

(Total for Question 14 is 4 marks)

1st

15

D

$3x + 2$

95°

A

75°

10°

10°

85°

C

$3x - 8$

85°

B

A, B, C and D are points on the circumference of a circle.
$ABCD$ is a trapezium with AB parallel to DC.

$AB = AC$

Work out the size of angle DAC.
You must show all your working.

[Reasons not required in this Q]

$3x + 2 + 3x - 8 = 180°$

Opposite angles in a cyclic quadrilateral add to $180°$

$6x - 6 = 180$

$6x = 186$

$x = 31°$

$3(31) + 2 = 95$

$3(31) - 8 = 85$

Angle ACB = Angle ABC = 85

Angle CAB = 10° Angles in a triangle add to 180°
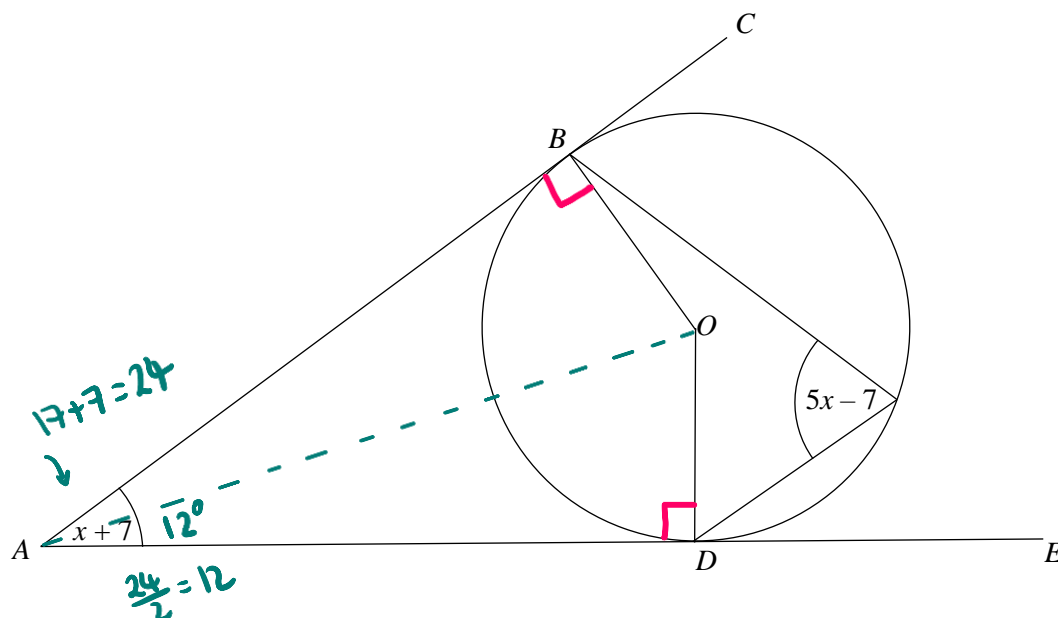
Angle DCA = Angle CAB alternate angles are equal

Angle DAC = 75° Angles in a triangle add to 180°

75 °

(Total for Question 15 is 5 marks)

1st

13

16

B and *D* are points on the circumference of a circle, centre *O*.
ABC and *ADE* are tangents to the circle.

[Reasons not required in this Q]

(a) Work out value of *x*.
You must show all your working.

Angle $BOD = 2(5x-7)$
(twice angle at circumference)

Angle $BOD = 180 - (x+7)$
(angles in quadrilateral add to 360°)

$10x - 14 = 180 - x - 7$
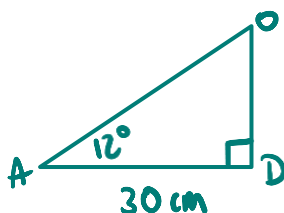$10x - 14 = 173 - x$
$11x = 187$
$x = 17$

$x =$ ....**17**........ °

(3)

AD = 30 cm

(b) Work out the radius of the circle.
Give your answer to three significant figures.

$BAD = 24°$
$OAD = 12°$

$\tan(12) = \dfrac{OD}{30}$

$OD = 30\tan(12)$

........**6.38**........ cm

(3)

(Total for Question 17 is 6 marks)

10

Annotations on diagram:
$17 + 7 = 24$
$\dfrac{24}{2} = 12$
$5x - 7$
$x + 7$
$12°$