Conditional Probability

SCAN ME

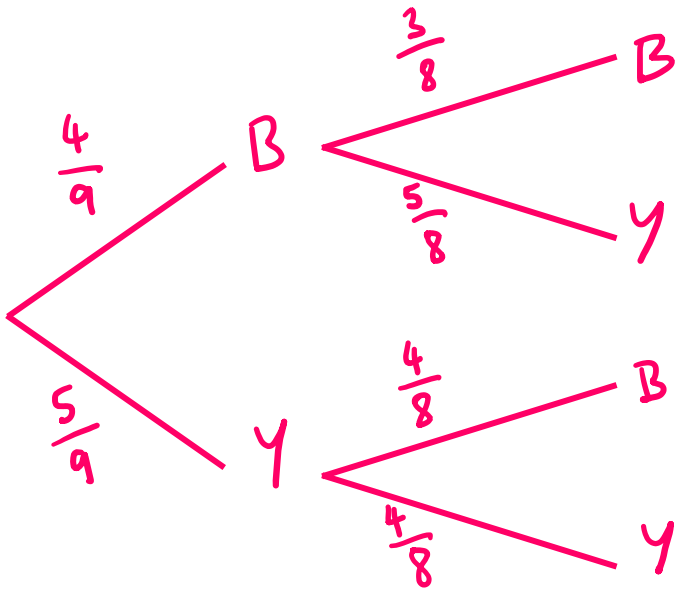
REVISE THIS TOPIC

Simplified answers are ok too!

1 There are 4 blue counters and 5 yellow counters in a bag.

Two counters are taken at random from the bag **without** replacement.

Work out the probability that both counters taken are the **same** colour. **[4 marks]**



$P(B,B) = \frac{4}{9} \times \frac{3}{8}$

$= \frac{12}{72}$

$P(Y,Y) = \frac{5}{9} \times \frac{4}{8}$

$= \frac{20}{72}$

$\frac{12}{72} + \frac{20}{72} = \frac{32}{72}$

Answer $\frac{32}{72}$

4

2 In a class there are 32 students.
13 of the students do higher tier maths and the other students do foundation.

Two of the students are selected at random from the class.

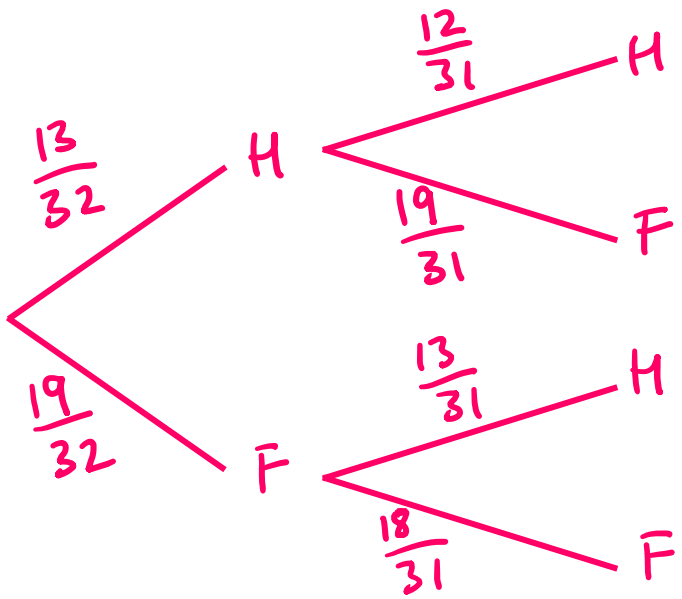
2 (a) Work out the probability that both students do higher tier maths. **[2 marks]**

$\frac{13}{32} \times \frac{12}{31} = \frac{156}{992}$

Answer $\frac{156}{992}$

2 (b) Show that

P(exactly one of the selected students does higher tier maths) < 50% **[4 marks]**



$P(H, F) = \frac{13}{32} \times \frac{19}{31}$ $\quad$ $P(F, H) = \frac{19}{32} \times \frac{13}{31}$

$= \frac{247}{992}$ $\quad$ $= \frac{247}{992}$

$\frac{247}{992} + \frac{247}{992} = \frac{494}{992} = 49.8\%$

3 Jamal's fruit bowl contains

5 apples
3 bananas
3 oranges

On Saturday and Sunday he eats one piece of fruit from the bowl.
Assume he chooses the fruit to eat randomly.

3 (a) Work out the probability that on Saturday he picks a banana and then on Sunday he picks an apple. **[2 marks]**

$P(B, A) = \frac{3}{11} \times \frac{5}{10}$

Answer $\frac{15}{110}$

3 (b) Work out the probability that both pieces of fruit chosen are the same fruit. **[4 marks]**

$P(A, A) = \frac{5}{11} \times \frac{4}{10}$

$= \frac{20}{110}$

$P(B, B) = \frac{3}{11} \times \frac{2}{10}$

$= \frac{6}{110}$

$P(O, O) = \frac{3}{11} \times \frac{2}{10}$

$= \frac{6}{110}$

$\frac{20}{110} + \frac{6}{110} + \frac{6}{110}$

Answer $\frac{32}{110}$

12

Turn over ►

4 In a café there are 3 slices of chocolate cake and 5 slices of vanilla sponge cake.

Two customers visit the café and each randomly buys one slice of cake.

4 (a) Work out the probability **at least** one of the customers had chocolate cake.

[4 marks]

Consider no chocolate

$P(V,V) = \frac{5}{8} \times \frac{4}{7}$

$= \frac{20}{56}$

$P(\text{at least one C}) = 1 - P(\text{no C})$

$= 1 - \frac{20}{56}$

Answer $\frac{36}{56}$

4 (b) A different café has 5 chocolate cake slices and 6 strawberry cheesecake slice. Each customer that enters also randomly buys once slice of cake.

Work out the probability that the first 3 customers all buy strawberry cake slices.

[2 marks]

$P(S,S,S) = \frac{6}{11} \times \frac{5}{10} \times \frac{4}{9}$

$= \frac{120}{990}$

Answer $\frac{120}{990}$

5 Beth's pencil case contains

4 red pens
5 green pens
7 black pens

Beth takes 3 pens at random from her pencil case.

5 (a) Work out the probability that all 3 pens the same colour. **[4 marks]**

$P(R,R,R) = \frac{4}{16} \times \frac{3}{15} \times \frac{2}{14} = \frac{24}{3360}$

$P(G,G,G) = \frac{5}{16} \times \frac{4}{15} \times \frac{3}{14} = \frac{60}{3360}$

$P(B,B,B) = \frac{7}{16} \times \frac{6}{15} \times \frac{5}{14} = \frac{210}{3360}$

$\frac{24}{3360} + \frac{60}{3360} + \frac{210}{3360}$

Answer $\frac{294}{3360}$

5 (b) The first pen chosen is black.
Work out the probability that all three pens are different colours. **[3 marks]**

Black removed so 4R, 5G, 6B

$P(R,G) = \frac{4}{15} \times \frac{5}{14} = \frac{20}{210}$

$P(G,R) = \frac{5}{15} \times \frac{4}{14} = \frac{20}{210}$

$\frac{20}{210} + \frac{20}{210} = \frac{40}{210}$

Answer $\frac{40}{210}$

13

6 There are 9 identical boxes on a gameshow.
6 of the boxes are empty and the remaining 3 boxes contain different prizes.
A gameshow contestant must select 3 boxes that will all be opened.

6 (a) Show that the probability that the contestant wins all 3 prizes is greater than 1% **[2 marks]**

$P(W,W,W) = \frac{3}{9} \times \frac{2}{8} \times \frac{1}{7}$

$= \frac{6}{504}$ (1.19%)

6 (b) Work out the probability that the contestant wins at least one prize. **[4 marks]**

Consider no prizes

$P(L,L,L) = \frac{6}{9} \times \frac{5}{8} \times \frac{4}{7}$

$= \frac{120}{504}$ $1 - \frac{120}{504}$

Answer $\frac{384}{504}$

6 (c) Work out the probability that the contestant wins **exactly one** prize. **[4 marks]**

$P(W,L,L) = \frac{3}{9} \times \frac{6}{8} \times \frac{5}{7} = \frac{90}{504}$

$P(L,W,L) = \frac{6}{9} \times \frac{3}{8} \times \frac{5}{7} = \frac{90}{504}$

$P(L,L,W) = \frac{6}{9} \times \frac{5}{8} \times \frac{3}{7} = \frac{90}{504}$

$\frac{90}{504} + \frac{90}{504} + \frac{90}{504} = \frac{270}{504}$

Answer $\frac{270}{504}$

7 Students who arrive late to school must enter through the late gate.

P(the **first student** to arrive late is in Year 11) = 0.4

For all students after the first student to arrive,

If the previous student was in Year 11, P(this student is in Year 11) = 0.75
If the previous student was not in Year 11, P(this student is in Year 11) = 0.1

7 (a) Work out the probability that the first student is in Year 11 and the second is not.

[2 marks]

P(Y11, not Y11) = 0.4 × 0.25
= 0.1

Answer 0.1

7 (b) Work out the probability **exactly two** of the first three students are in Year 11.

[4 marks]

P(Y11, Y11, not Y11) = 0.4 × 0.75 × 0.25
= 0.075

P(Y11, not Y11, Y11) = 0.4 × 0.25 × 0.1
= 0.01

P(not Y11, Y11, Y11) = 0.6 × 0.1 × 0.75
= 0.045

0.075 + 0.01 + 0.045 = 0.13

Answer 0.13

16

8 A car showroom has 5 blue cars and 3 white cars for sale.

Assume that each of the cars has an equal chance of being sold.

On Monday 3 of the cars from the showroom are sold.

8 (a) Work out the probability that all three cars sold are the same colour.

[4 marks]

$P(B,B,B) = \frac{5}{8} \times \frac{4}{7} \times \frac{3}{6} = \frac{60}{336}$

$P(W,W,W) = \frac{3}{8} \times \frac{2}{7} \times \frac{1}{6} = \frac{6}{336}$

$\frac{60}{336} + \frac{6}{336} = \frac{66}{336}$

Answer $\frac{66}{336}$

8 (b) Work out the probability that there is only one white car left in the showroom.

[4 marks]

$P(W,W,B) = \frac{3}{8} \times \frac{2}{7} \times \frac{5}{6} = \frac{30}{336}$

$P(W,B,W) = \frac{3}{8} \times \frac{5}{7} \times \frac{2}{6} = \frac{30}{336}$

$P(B,W,W) = \frac{5}{8} \times \frac{3}{7} \times \frac{2}{6} = \frac{30}{336}$

$\frac{30}{336} + \frac{30}{336} + \frac{30}{336} = \frac{90}{336}$

Answer $\frac{90}{336}$

9 There are 5 green counters and 2 yellow counters in a bag.

One counter is taken at random from the bag.
Two counters of the opposite colour are then added to the bag.

A final counter is taken at random from the bag.

9 (a) Work out the probability that both counters removed from the bag are yellow.

[2 marks]

$P(Y,Y) = \frac{2}{7} \times \frac{1}{8}$

$= \frac{2}{56}$

Answer $\frac{2}{56}$

9 (b) Work out the probability that there are more green counters left in the bag than there are yellow counters left in the bag.

[4 marks]

$P(Y,Y) = \frac{2}{56}$

$P(G,Y) = \frac{5}{7} \times \frac{4}{8} = \frac{20}{56}$

$P(Y,G) = \frac{2}{7} \times \frac{7}{8} = \frac{14}{56}$

$\frac{2}{56} + \frac{20}{56} + \frac{14}{56}$

Answer $\frac{36}{56}$

14

10 Tim's toy box contains some cars and some lorries that are red, white or blue.

	Car	Lorry	
Red	3	1	4
White	6	3	9
Blue	5	2	7

Tim takes two toys from his toy box at random.

10 (a) Work out the probability that one toy is a car and the other is a lorry. **[4 marks]**

$P(C, L) = \frac{14}{20} \times \frac{6}{19} = \frac{84}{380}$

$P(L, C) = \frac{6}{20} \times \frac{14}{19} = \frac{84}{380}$

$\frac{84}{380} + \frac{84}{380} = \frac{168}{380}$

Answer $\frac{168}{380}$

10 (b) Work out the probability that both toys are different colours. **[4 marks]**

$P(R, R) = \frac{4}{20} \times \frac{3}{19} = \frac{12}{380}$

$P(W, W) = \frac{9}{20} \times \frac{8}{19} = \frac{72}{380}$

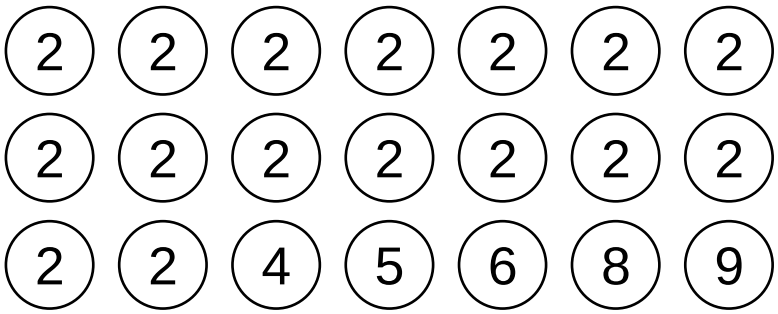
$P(B, B) = \frac{7}{20} \times \frac{6}{19} = \frac{42}{380}$

$+ \quad \frac{126}{380}$

$P(\text{different colour}) = 1 - \frac{126}{380}$

Answer $\frac{254}{380}$

11 A bag contains the following numbered counters.



Two of the counters are taken from the bag at random.

Work out the probability that the product of the numbers on the two counters is a square number.

[5 marks]

$P(2,2) = \frac{16}{21} \times \frac{15}{20} = \frac{240}{420}$

$P(2,8) = \frac{16}{21} \times \frac{1}{20} = \frac{16}{420}$

$P(8,2) = \frac{1}{21} \times \frac{16}{20} = \frac{16}{420}$

$P(4,9) = \frac{1}{21} \times \frac{1}{20} = \frac{1}{420}$

$P(9,4) = \frac{1}{21} \times \frac{1}{20} = \frac{1}{420}$

$\frac{240}{420} + \frac{16}{420} + \frac{16}{420} + \frac{1}{420} + \frac{1}{420} = \frac{274}{420}$

Answer $\frac{274}{420}$

/13